OPTIMAL LAND MANAGEMENT WITH MULTIPLE CROPS

Graeme J. Doole[1,2,3] and David J. Pannell[1,2]

[1]School of Agricultural and Resource Economics, Faculty of Natural and Agricultural Sciences, University of Western Australia, 35 Stirling Highway, Crawley, Western Australia 6009.

[2]Cooperative Research Centre (CRC) for Plant-Based Management of Dryland Salinity, University of Western Australia, 35 Stirling Highway, Crawley, Western Australia 6009.

[3]CRC for Australian Weed Management, Waite Road, Urrbrae, PMB 1, Waite Campus, Glen Osmond, South Australia 5064.

Abstract. This paper examines the optimal management of agricultural land through the use of non-crop inputs, such as fertiliser, and land uses that either degrade or restore productivity. We demonstrate the need to consider the relative total asset value of alternative crops over time. It is shown that higher prices for crops that degrade the resource base should motivate the use of short rotations with a remedial phase. An inability of land markets to reflect differences in resource quality and low capital malleability promote greater degradation. However, substitution of complementary effects through input usage may help to sustain productivity. These factors are discussed in the context of crop sequence management in Western Australian cropping systems.

Keywords. Crop sequences, land degradation, optimal switching.

JEL classification codes. Q15; Q24.

1. Introduction

Crop sequences are utilised extensively in order for agricultural production to benefit from complementary relationships between land uses. Direct benefits of rotations are the lowering of risk and the smoothing of input demand (Wu, 1999; Annetts and Audsley, 2002). Indirect benefits are those that influence profit by increasing the production of subsequent crops. Examples are the interruption of pest and disease cycles, the reduction of soil erosion, nitrogen fixation by legumes, enhanced soil structure, management of crop residues, and weed management (El-Nazer and McCarl, 1986; Pannell, 1995; Wu and Babcock, 1998). Aggregation of indirect benefits provides an indication of the productivity of the land resource. This is analogous to a form of capital stock that partly determines crop production.

McConnell (1983) formalised the relationship between capital theory and the base productivity of agricultural land. In this model, conservation reduced current production and degradation could not be offset through the addition of non-soil inputs, such as fertiliser (McConnell, 1983; Barrett, 1991). Barbier (1990) extended this framework to incorporate "productive" and "ameliorative" inputs. The first increased both output and soil loss, for example deep cultivation. The second decreased erosion but did not affect crop production directly, for example the construction of terraces. Determining the optimal usage of the former is similar to renewable resource exploitation, while the latter resembles traditional investment theory (Clarke, 1992; LaFrance, 1992). Links to capital theory are even stronger when ameliorative practices enter as stock variables and investment therefore has a lasting impact on conservation (Grepperud, 1997).

The single crop approach adopted in these papers disregards complementary effects between agricultural practices. However, such complementary effects can often be important in reality (Pannell, 1987; Orazem and Miranowski, 1994). Indirect effects may be incorporated by identifying the optimal allocation of a given area of land among crop and non-crop land uses, such as pasture, at each point in time (Goetz, 1997). However, aggregation of the impacts of land use on productivity across an entire farm provides a coarse approximation of their value. A more precise examination requires analysis at the field level (Burt, 1981), particularly since spatial heterogeneity in land quality is a characteristic of many agricultural systems (Howitt, 1995). In this case, it is appropriate to analyse crops as discrete choices rather than proportions.

Rotations between agricultural practices that degrade or restore land quality have been studied previously (Willassen, 2004). However, non-crop inputs, such as fertiliser or herbicide, were omitted from this analysis, as the primary focus was the fallow-cultivation cycle of traditional agriculture. This paper extends that work through the inclusion of multiple land uses and non-crop inputs that increase base productivity. An important result is that increases in the price of output for product from degrading crops should stimulate frequent rotation with an enterprise that restores base productivity. Key findings are discussed in relation to single crop models and the management of crop sequences in Western Australian agricultural systems.

The switching problem and necessary conditions for an optimal solution are presented in the next section. The model is based on the general framework of Doole et al. (2005). Implications for optimal land management under single and multiple crops are outlined in Section 3 and Section 4 respectively. Conclusions are presented in the final section.

2. A switching model for investigation of land degradation

This section contains a description of an optimal switching model for the analysis of multiple crops and presents necessary conditions for its solution. Assume that a producer must determine the most profitable use of a field between t_0 and t_2. A given enterprise, *i*=1, is active at the outset. However, the farmer may decide to switch to a successive regime, *i*=2, at any time, t_1, between these endpoints. The moment before the switch occurs is denoted t_1^- and the moment after the switch has occurred is t_1^+. Regime *i* is therefore active over the closed interval $t=[t_{i-1}^+, t_i^-]$.

The framework could incorporate *n* switches but only one is incorporated here for clarity of exposition. Switching must occur before the sale of the farm at t_2 as endogenous determination of total switching moments has proven intractable. Although an abstraction, bias is reduced through placing no constraints on regime length, as long as $t_0 < t_1 < t_2$.

It is assumed that the quality of a fixed area of land in terms of agricultural production may be described by a composite index denoted by a single state variable, *x(t).* Investment in land capital is represented by an increase in this index while disinvestment causes a decline.

Productivity may be manipulated through the intensity of management inputs, the control variable denoted as $u_i(t)$. Controls are defined as continuous functions for generality but in most contexts are likely to be discrete practices. Subscription by regime index permits the set of management inputs (U^i, where $u_i(t) \in U^i$) to differ for each land use. For example, a herbicide may control weeds effectively in crop 1 but harm crop 2 significantly. In this case, U^1 may contain this herbicide but U^2

would not.

Productivity is also influenced through crop choice. Rates of degradation and renewal for each land use i are described through motion functions, $f_i(x(t), u_i(t))$. The units of measurement for the motion functions will depend on the definition of the state variable. A regime may be either degrading ($f_{\deg}(\cdot) < 0$) or restoring ($f_{res}(\cdot) > 0$) of land quality. Examples are wheat crops that degrade soil structure and pasture legumes that restore soil nitrogen and organic matter. The case where land is not affected through crop choice is ignored.

It is assumed that the use of an input has a net increasing impact on crop yield. Application increases base productivity, therefore $[f_i(\cdot)]_u > 0$, where $[\cdot]_u$ denotes the derivative of the function in square brackets with respect to (w.r.t) the subscripted term. An increase in productivity following input use will augment crop yield, $y_i(x(t))$, through the intuitive assumption, $[y_i(\cdot)]_x > 0$. An example is nitrogen fertiliser that increases crop yield through increasing soil nitrogen. Other types of input may be more applicable to certain problems. For example, “productive” inputs may be used to investigate the management of systems where practices, such as deep cultivation, increase output but degrade base productivity. These may be easily incorporated in this framework with adjustment of the relevant relationships. The critical difference between productive inputs and those analysed in this paper is that the latter allow intensification to occur without degradation.

A state vector could represent base productivity in place of a composite index. This vector would contain a number of individual determinants of production, each with its own motion equation and associated control set. For example, state-transition equations representing a weed population and total soil nitrogen could be included.

The control set for the former could incorporate different intensities of herbicide while that for the latter could involve alternative levels of nitrogen fertiliser. The use of a composite index is retained for broader relevance and to avoid problems associated with dimensionality.

The initial level of land quality is denoted $x(t_0) = x_0$. The state trajectory is determined by,

$$\dot{x}(t) = f_i(x(t), u_i(t)), \tag{1}$$

for i={1,2}. This is continuous but non-differentiable at the switching time, t_1.

A continuous profit function $\pi_i(x(t), u_i(t), t)$ is defined for each regime i,

$$\pi_i(x(t), u_i(t), t) = \int_{t_{i-1}^{+}}^{t_i^{-}} e^{-\delta t}\left(p_i y_i(x(t)) - c_i(x(t), u_i(t))\right)dt, \tag{2}$$

where $e^{-\delta t}$ is a discount factor with δ as the discount rate, p_i is the price per unit of output, $y_i(x(t))$ is output, and $c_i(x(t), u_i(t))$ is the cost of inputs. As defined earlier, $[y_i(\cdot)]_x > 0$. Costs are assumed to increase as land quality declines due to decreases in the effectiveness of inputs. For example, more expensive cultural treatments are required when a herbicide-resistant weed population develops (Pannell and Zilberman, 2001; Monjardino et al., 2004). Therefore, $[c_i(\cdot)]_x < 0$. However, this assumption may be relaxed with little effect on the following discussion. In addition, inputs are costly so $[c_i(\cdot)]_u > 0$.

Land has a salvage value defined through the function $e^{-\delta t_2} h(x(t_2))$. This is assumed to increase with the productivity of land, so $[e^{-\delta t_2} h(x(t_2))]_x > 0$. Moving from one land use to another incurs a switching cost, $e^{-\delta t_1^-} s(x(t_1^-))$. This increases with declining land quality, therefore $[e^{-\delta t_1^-} s(x(t_1^-))]_x < 0$. An example is pasture establishment for which costs increase as weed populations burgeon. The latter assumption may not be relevant for certain problems. It may consequently be disregarded in such situations with little effect on the following discussion.

The producer's problem for *i*={1,2} is,

$$\max_{u_i, t_i} J = \int_{t_0}^{t_1^-} \pi_1(x(t), u_1(t), t)dt - e^{-\delta t_1^-} s(x(t_1^-)) + \int_{t_1^+}^{t_2} \pi_2(x(t), u_2(t), t)dt + e^{-\delta t_2} h(x(t_2)), \tag{3}$$

subject to,

$$\dot{x}(t) = f_i(x(t), u_i(t)), \text{ and,} \tag{4}$$

$$x(t_0) = x_0. \tag{5}$$

This problem incorporates two standard free-time optimal control problems with terminal value functions. Solution is complicated because the management of the first regime influences the second through the state variable and the optimal switching time must be endogenously determined. Necessary conditions for the solution of a similar model have been derived (Amit, 1986). However, that formulation does not include a terminal value function. Solution therefore requires Theorem 1.

Theorem 1 (Doole et al., 2005). Let $(x^*(t), u_i^*(t), t_i^*)$ for *i*={1,2} denote the trajectory that maximises *J* in (3) subject to the constraints (4) and (5). This is the *optimal trajectory*. A Hamiltonian function for each regime *i* is defined as,

$$H_i(x(t), u_i(t), \lambda_i(t), t) = \pi_i(x(t), u_i(t), t) + \lambda_i(t) f_i(x(t), u_i(t), t). \tag{6}$$

Under the optimal trajectory there exists a vector of piecewise continuous adjoint functions, $\lambda = [\lambda_1(t), \lambda_2(t)]$, that each satisfies, over the closed interval t=[t_{i-1}^+, t_i^-],

$$\dot{\lambda}_i(t) = -\frac{\partial H_i(x(t), u_i(t), \lambda_i(t), t)}{\partial x(t)}. \qquad (7)$$

The optimal control function within each land use i must obey,

$$\frac{\partial H_i(x(t), u_i(t), \lambda_i(t), t)}{\partial u_i(t)} = 0. \qquad (8)$$

The following conditions must be satisfied at the final time,

$$H_2(x(t), u_2(t), \lambda_2(t), t)\Big|_{t_2} + \frac{\partial e^{-\delta t_2} h(x(t_2))}{\partial t_2} = 0 \text{, and,} \qquad (9)$$

$$\lambda_2(t_2) = \frac{\partial e^{-\delta t_2} h(x(t_2))}{\partial x(t_2)}. \qquad (10)$$

The Hamiltonian functions for each regime at the switching time t_1 must obey,

$$H_1(x(t), u_1(t), \lambda_1(t), t)\Big|_{t_1^-} - \frac{\partial e^{-\delta t_1^-} s(x(t_1^-))}{\partial t_1^-} = H_2(x(t), u_2(t), \lambda_2(t), t)\Big|_{t_1^+}. \qquad (11)$$

In addition, the adjoint variables at the switching time t_1 must satisfy the boundary condition,

$$\lambda_1(t_1^-) + \frac{\partial e^{-\delta t_1^-} s(x(t_1^-))}{\partial x(t_1^-)} = \lambda_2(t_1^+). \qquad (12)$$

Conditions (6) to (10) are consistent with the solution of a standard free-time optimal control problem with a salvage value (Kamien and Schwartz, 1991). Conditions (11) and (12) are not. These collectively specify the relationships that must hold at the switching time. The first outlines that it is beneficial to switch to the successive regime when it is more profitable to do so and incur switching costs than remain in the active land use. Similarly, the second states that it is optimal to switch from one agricultural practice to another when the marginal value of renewal or degradation matches that within the next regime.

3. Optimal management of individual crops

This section examines the optimal management of agricultural land within individual regimes. This outlines the implications of salvage value for effective stewardship and provides a foundation for the discussion of switching dynamics that follows.

The Hamiltonian function for each regime *i* is,

$$H_i(x(t),u_i(t),\lambda_i(t),t) = e^{-\delta t}\left(p_i y_i(x(t)) - c_i(x(t),u_i(t))\right) + \lambda_i(t) f_i(x(t),u_i(t)). \tag{13}$$

The Hamiltonian function ($H_i(\cdot)$) represents the total capital value of regime *i* and consists of two terms. The first is discounted profit. The second is the user benefit or user cost associated with current management, $\lambda_i(t) f_i(\cdot)$. This is the total gain or loss in future profit from time *t* to the end of the regime following an increase or decrease in base productivity. User benefit/cost involves two terms. The shadow price of renewal or degradation ($\lambda_i(t)$) reflects the effect of a change in base productivity at time *t* on profits earned over the remainder of the regime's duration. The second term is the motion function, as defined earlier.

Together with the state equation (4) and the initial condition (5), optimal trajectories within a given land use must satisfy,

$$\frac{\partial H_i(\cdot)}{\partial u_i(t)} = -e^{-\delta t}[c(x(t),u_i(t))]_u + \lambda_i(t)[f_i(x(t),u_i(t))]_u = 0, \text{ and,} \tag{14}$$

$$\dot{\lambda}_i(t) = -\frac{\partial H_i(\cdot)}{\partial x(t)} = -e^{-\delta t}\left(p_i[y_i(x(t))]_x - [c_i(x(t), u_i(t))]_x\right) - \lambda_i(t)[f_i(\cdot)]_x . \qquad (15)$$

The first equation, (14), identifies that inputs will be used up to the point where their marginal cost ($[c(\cdot)]_u$) is equal to their marginal benefit ($\lambda_i(t)[f_i(\cdot)]_u$). Marginal benefit consists of the physical relationship between input application and the rate of degradation/renewal ($[f_i(\cdot)]_u$) multiplied by the marginal value of this change in base productivity ($\lambda_i(t)$). This specification contrasts that presented with the inclusion of "productive" inputs that enter the production function of crops directly (Barbier, 1990; Clarke, 1992; LaFrance, 1992). In this case, the marginal benefit of input use is marginal value product ($p_i[y_i(x(t), u_i(t))]_u$, where $[y_i(\cdot)]_u > 0$), the value of marginal output accruing to input application.

The second equation, (15), identifies that the rate of depreciation/appreciation of land capital is the total of its marginal contribution to direct profits and capital investment under optimal management (Dorfman, 1969). Greater insight can be gained through integration of (15) for the *second* regime,

$$\lambda_2(t) = e^{-\delta s}\int_s^{t_2} e^{-\alpha}\left(p_2(\cdot)[y_2(\cdot)]_x - [c_2(\cdot)]_x\right)dt + e^{-\alpha}[h(x(t_2))]_x , \qquad (16)$$

where $\alpha = (\delta + [f_2(\cdot)]_x)(t - s)$. Equation (16) identifies that the shadow price of a change in base productivity at time s ($s > t_1$) is the present value of the marginal profit earned between the present and the terminal time. Marginal profit is discounted by δ. In addition, it is discounted or compounded by the rate, $[f_i(\cdot)]_x$, at which degradation or renewal change with land quality. This term resembles the marginal growth rate of biological assets that must be weighted with the discount rate in the determination of an equilibrium stock and harvest level in bioeconomic fishery models (Clark, 1990).

The rate of degradation under a given crop may increase as land quality declines. For instance, the onset of soil salinisation is promoted when a rising saline water table reaches a threshold depth because its rise to the surface is enhanced by capillary action (McIntyre, 1982). This implies $\left[f_i(\cdot)\right]_x < 0$ and is analogous to a greater discount rate. Alternatively, it is generally easier for crop or pasture plants to compete with lower weed populations (Cousens, 1985; Taylor, 1987). This, in contrast, implies $\left[f_i(\cdot)\right]_x > 0$ under a land use that restores productivity and counteracts discounting. Either relationship could hold in the opposite direction. That is, degradation may slow and/or the rate of renewal may increase as base productivity declines. However, such counter-examples are less intuitive and are therefore disregarded, although their implications may be easily examined in terms of the following discussion.

Degradation ($\dot{x}(t) < 0$) reduces profit in the second regime through decreasing yield ($[y_2(\cdot)]_x$>0), increasing input costs ($[c_2(\cdot)]_x$<0), and imposing user costs ($\lambda_2(t) f_2(\cdot) < 0$). A lower shadow price implies a decrease in the future profitability of this regime. This reduces user cost and thus encourages more intensive resource use. The causes of such a reduction are apparent from (16). The discount rate represents the opportunity cost of capital. Higher returns elsewhere in the economy therefore motivate degradation (McConnell, 1983). As noted earlier, degradation rates may increase as land quality declines. This will promote exploitation so that discounting has a lesser effect on the profits accruing to degradation. Declines in marginal profit, such as through lower prices, will also decrease the shadow price.

If the terminal regime restores agricultural productivity ($\dot{x}(t) > 0$) then $\lambda_2(t) f_2(\cdot)$

instead represents a user benefit. A lower shadow price ($\lambda_2(t)$) will decrease the magnitude of user benefit and therefore decrease the incentive to retain this land use. In line with the results for a phase that degrades the resource base, this also occurs with a higher discount rate and lower marginal profit. However, these effects are reduced through rates of renewal that are augmented with increasing resource productivity, i.e. $[f_i(\cdot)]_x > 0$.

The producer will sell the land at the point where continuing farming of the last regime is unprofitable. Here the following relationship, consistent with (9), holds,

$$H_2(\cdot)\Big|_{t_2} - \delta e^{-\delta t_2} h(x(t_2)) + e^{-\delta t_2}[h(x(t_2))]_x \dot{x}(t) = 0 . \qquad (17)$$

The first term (the Hamiltonian function for the second regime evaluated at t_2) represents the marginal value of extending the length of the final regime. The second and third terms are the rate at which the discounted salvage value of the farm changes with adjustment of the terminal time. The sum of these three terms must be zero at the optimal time of sale; otherwise it is profitable to continue management.

Prolonging the planning horizon will have two effects on the resale value of the farm. These are reflected in the second and third terms in (17). First, salvage value will decrease through discounting. Second, it will decrease (increase) with retention of the last regime if this regime degrades (restores) land quality. However, the last factor declines in importance with decreases in the degree to which land markets reflect differences in productivity.

The price of agricultural land should reflect its expected long-term profitability under perfect information (Just and Miranowski, 1993). Optimal management consequently requires explicit consideration of current actions on the resale value of the farm (see equations (16) and (17)). However, this may not occur if there are information failures, notions of bequest are weak, or capital markets do not clear (McConnell, 1983). Incentives for conservation will be reduced if land markets do not properly account for differences in productivity (Clarke, 1992; Goetz, 1997). Suboptimal levels of exploitation will consequently be utilised, the degree of disinvestment in land capital depending on the extent to which the salvage value term is sensitive to degradation of the base resource. This will be highest in the extreme case where $h(x(t_2))$ is independent of land quality as producers will have no incentive to conserve the land resource in order to obtain a higher terminal value.

There is conflicting evidence on the degree to which the quality of agricultural land is capitalised in property markets. Some evidence supports a connection between resource conservation and increased land prices (King and Sinden, 1989). However, anecdotal evidence suggests that strategies to mitigate soil salinisation in Western Australia are hampered through the failure of farm prices to consider their value. Survey work identifies that producers would pay significantly less for cropping land where greater levels of herbicide resistance are evident (Llewellyn et al., 2002). Nonetheless, capitalisation is hindered through costs of acquiring suitable data and monetary benefit accruing to the maintenance of asymmetric information.

4. Optimal management of multiple crops

This section focuses on the analysis of crop sequences utilising the framework described in Section 2. Determining the optimal rate of exploitation or investment across the relevant planning horizon is the sole consideration if a single crop exists. However, intertemporal planning requires simultaneous consideration of the relative (potential) profitability of the successive regime if a planting alternative is available.

Suppose that it is presently more profitable to remain in the first enterprise. The Left Hand Sides (LHSs) of switching conditions (11) and (12) will consequently be greater than the Right Hand Sides (RHSs). Over time, any change in the level of the state variable under the active regime will modify both its own marginal value and that of the next regime. Adjustment will continue until the condition holds with equality, beyond which it is more profitable to switch than remain in the first regime.

Detailed analysis is possible from manipulation of system equations and either (11) or (12). For example, substitute the definition of the Hamiltonian function in (13) into the switching condition (11). Multiply this equation by $e^{\delta t}$ and take the derivative w.r.t *t*. Utilising the motion equation (1) and the optimal control condition (8), the switching condition that holds at t_1 is,

$$\begin{array}{l} \left\{ f_1(\cdot)\left(p_1(\cdot)[y_1(\cdot)]_x - [c_1(\cdot)]_x + e^{\delta t}\lambda_1(t)\left(\delta + \dot{\lambda}_1(t) + [f_1(\cdot)]_x\right)\right)\right\}\Big|_{t_1^-} = \\ \left\{ f_2(\cdot)\left(p_2(\cdot)[y_2(\cdot)]_x - [c_2(\cdot)]_x + e^{\delta t}\lambda_2(t)\left(\delta + \dot{\lambda}_2(t) + [f_2(\cdot)]_x\right)\right)\right\}\Big|_{t_1^+} \end{array} . \qquad (18)$$

The marginal value of the active land use (the value of the LHS of (18)) will decline over time if this regime degrades the base resource. This occurs through $f_1(\cdot)$ which is defined as $f_{\deg}(\cdot) < 0$ with a degrading enterprise. Declining land quality will influence the marginal value of the regime through decreasing yield ($[y_1(\cdot)]_x$) and increasing input ($[c_1(\cdot)]_x$) and switching costs ($[s(\cdot)]_x$). Alternatively, the LHS of (18) will increase over time if the first regime restores land quality. In this situation, $f_1(\cdot)$ will be replaced with $f_{res}(\cdot) > 0$. Increases in productivity will increase yield and decrease both input and switching costs.

Changes in base productivity within a given regime influence its future profitability, as well as the relative marginal value of the successive land use (if one is defined). The current value multiplier ($e^{\delta t}\lambda_i(t)$) is the marginal value of a change in $x(t)$ evaluated at time *t*. Multiplication by the motion function ($f_i(\cdot)$) gives the current value of user cost/benefit, the decrease/increase in future profitability attributable to degradation/restoration. User cost/benefit will be compounded at the rate of return, δ, given its definition as a current value but will also be affected through depreciation/appreciation of land capital ($\dot{\lambda}_i(t)$) and the rate at which degradation or renewal changes with land quality ($[f_i(\cdot)]_x$).

Perturbation of parameters within switching conditions provides a heuristic means of analysing the optimal management of crop sequences. Switching conditions (11) and (12) will hold with equality along the optimal trajectory. However, consider a perturbation in the marginal value accruing to a change in base productivity in regime *i*, with all other factors held constant. The consequent increase/decrease in the profitability of this land use, relative to its successor (*i*+1), should delay/hasten switching under optimal management. This is because it is more/less profitable to remain in the active regime.

For example, a higher price for the first regime, with all other factors held constant, will increase the marginal value of output and the shadow price of renewal if this crop invests in land quality. Optimal management requires delayed switching because it is more profitable to remain in the active land use. In contrast, a higher price for a degrading first regime, *ceteris paribus*, will increase the value of yield reductions ($\dot{x}(t)p_1(\cdot)[y_1(\cdot)]_x < 0$ in (18)) lost through disinvestment in land quality. Switching should therefore occur sooner under optimal management. These findings extend similar propositions in models that allocate land among different agricultural practices in each time period (Goetz, 1997) or do not include non-crop inputs (Willassen, 2004).

Higher prices generally motivate increases in the area of land planted to more valuable crops, in accordance with the law of supply. For instance, the higher profitability of cereal crops relative to livestock enterprises motivated extended cropping phases in Western Australia throughout the 1990s (Reeves and Ewing, 1993; Poole et al., 2002). The above results identify that increasing the use of a degrading regime following a price rise is suboptimal. In line with this argument, extended cropping phases throughout Australia have promoted extensive land degradation in terms of soil structure decline, herbicide resistance, and soil salinisation (Reeves and Ewing, 1993; Monjardino et al., 2004).

Although suboptimal, such a response is consistent with recommendations from applications of equilibrium whole-farm optimisation models in the farm planning literature, a predictable result given the necessary conditions for static profit maximisation in production economics. Omitting the dynamics of land degradation and its relationship with yield results in little discernment between degrading and renewing crops in these frameworks. Recommendations for greater degradation may consequently follow a price increase. Indeed, one author has identified this as "conventional wisdom... in the agricultural economics profession" (Clarke, 1992. p. 31). This discussion encourages a cautious approach to the application of static models when crop choice and base productivity are interrelated.

Suppose enterprises that degrade and restore base productivity are grown in rotation. In line with the above discussion, a price increase for the former should promote the adoption of a shorter regime. In addition, decreases in the unit price for product of the land use that restores land quality should stimulate earlier switching into the next phase. This is a degrading enterprise given the defined rotation. This argument implies that frequent rotation with enterprises that restore the productivity of land should be encouraged when higher relative prices are received for the degrading land use. This finding identifies the importance of utilising rotations with a deep-rooted perennial pasture, such as lucerne (*Medicago sativa*), to improve the long-term profitability of Western Australian agricultural systems. These pastures are of significant benefit given their ability to improve soil structure, prevent the development of herbicide resistance through permitting the use of integrated weed management, and reduce recharge to saline water tables through the creation of a dry soil buffer (Ward et al., 2002). This finding is of particular value given that it is converse to standard economic arguments and would not be identified in a single crop model.

Many single-crop analyses identify an indeterminate relationship between price and land degradation. A higher price may promote degradation through motivating greater use of productive inputs or encourage conservation through increasing the marginal benefit of soil conservation (Clarke, 1992; LaFrance, 1992; Grepperud, 1997). The optimal response of planting decisions to a higher output price in this paper depends on whether a crop restores or degrades base productivity. Nonetheless, a higher output price will promote greater investment in the base resource through increasing the marginal benefit of input application through the costate variable (see (14) and (16)).

Improving the cost-effectiveness of inputs ($[c_i(\cdot)]_x$) will decrease the marginal value of a regime that restores productivity, *ceteris paribus*. This should decrease the optimal length of a restoring phase. This follows more efficient substitution between a restoring land use and non-crop inputs that invest in base productivity. For example, cheaper nitrogen fertiliser may shorten the length of time that a legume phase is required.

In contrast, improving the efficiency of inputs available during a degrading land use, with all other factors held constant, will increase the marginal value of this regime. Switching should consequently be delayed under optimal management because of the availability of more efficient inputs to slow degradation. This is observable in Western Australian agricultural systems where technical innovation has motivated extended cropping sequences through the provision of economic substitutes for the indirect effects offered through a pasture ley. The development and adoption of grain legumes, such as lupins (*Lupinus spp.*) and faba beans (*Vicia faba*), permits biological nitrogen fixation without pasture establishment. The increased use of nitrogen fertiliser has also helped to reduce the value of pasture legumes (Angus, 2001). In addition, selective and knockdown herbicides have helped to substitute for the weed control benefits offered by a pasture phase. However, the optimality of the switching decision will depend on careful consideration of the effectiveness of substitution. For example, reliance on chemical control of weed populations during an extended cropping phase is likely to promote the development of herbicide resistance (Monjardino et al., 2004).

A decrease in marginal switching costs ($[s(x(t_1^-))]_x$) should extend the length of a degrading regime under optimal management. For example, more efficient techniques for crop establishment should motivate an extended cropping phase. In contrast, a decrease in marginal switching costs lessens the marginal benefit of renewal. A land use that restores base productivity should therefore terminate earlier. An example would be a reduction in the value accruing to integrated weed management during a pasture phase when herbicide-resistant weeds do not threaten effective crop establishment.

Switching costs will change solely through discounting if independent of the state variable. In this situation, the transition cost relationship could be denoted $e^{-\delta t_i} s$ and Equation (11) would include $[-e^{-\delta t_i} s]_{t_1^-} = \delta e^{-\delta t_i} s > 0$. This term is positive because extending the length of the active regime will reduce switching costs through discounting, thereby increasing dynamic profitability. An increased transition cost will increase the benefit accruing to discounting so that is optimal to extend the length of the active regime. Some Western Australian producers invested heavily in cropping machinery and removed infrastructure required for livestock management, such as fencing, following higher prices for crop product throughout the 1990s. In line with the above proposition, difficulties associated with disinvestment in capital (low capital malleability) has increased the cost associated with switching from the degrading cropping phase to the pasture enterprise. This has motivated extension of the degrading regime and therefore promoted greater disinvestment in the land resource.

The converse of this relationship suggests that an absence of switching costs should promote earlier switching. This is consistent with frequent rotation between cereal crops and annual legume pastures in traditional farming systems in southern Australia (Pannell, 1995). Switching costs are low in these systems because the annual pasture regenerates from hard seed and does not require expensive herbicide application for its removal. This finding contrasts that of Willassen (2004), who identified that an absence of switching costs should instead motivate continued adoption of a degrading phase. This conclusion seems less intuitive from that identified here but is consistent with the application of impulse control when transition costs are zero.

5. Conclusions

Dual consideration of non-crop inputs and complementary effects between land uses resolves a significant shortcoming in the analysis of land degradation. This addition brings such models closer to representing modern agricultural systems in which both non-crop inputs and indirect effects play important roles in maintaining farm profitability. A key result is that price increases for degrading crops should motivate more frequent rotation with agricultural practices that restore land quality. A second outcome is that non-crop inputs may be used to offset degradation although careful consideration is required if their long-term effectiveness is to be maintained. A third implication is that low capital malleability may promote degradation through increasing the cost of switching between alternative land uses. These factors demonstrate that consideration of multiple crops has important implications for optimal land management. These are of significance to the application of numerical frameworks that may provide more specific recommendations for given agricultural systems. To this end, the development of practicable methods for the numerical optimisation of switching systems is worthy of further attention.

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